Exploring CP violation with $B_s^0 \to \bar{D}^0 \phi$ decays

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We note that it is possible to determine the weak phase γ from the time dependent measurements of the decays $B_s^0(t)(\bar{B}_s^0(t)) \to \bar{D}^0 \phi$ without any hadronic uncertainties. These decays are described by the color suppressed tree diagrams and hence are free from the penguin pollutions. We further demonstrate that γ can also be extracted with no hadronic uncertainties from an angular analysis of corresponding vector vector modes, $B_s^0(t)((\bar{B}_s^0(t))) \to \bar{D}^{*0} \phi$. Although the branching ratios for these decay modes are quite small $\mathcal{O}(10^{-5} - 10^{-6})$, the strategies presented here appear to be particularly interesting for the "second generation" experiments at hadronic B factories.

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I. INTRODUCTION

Despite many attempts, CP violation still remains one of the most outstanding problems in particle physics [1–3]. The standard model (SM) with three generations provides a simple description of this phenomenon through the complex Cabibbo-Kabayashi-Maskawa (CKM) matrix [4]. Decays of B mesons provide a rich ground for investigating CP violation [5,6]. They allow stringent tests both for the SM and for studies of new sources of this effect. Within the SM, CP violation is often characterized by the so-called unitarity triangle [7]. Detection of CP violation and the accurate determination of the unitarity triangle are the major goals of experimental B physics [8]. Decisive information about the origin of CP violation in the flavor sector can be obtained if the three angles $\alpha(\equiv \phi_2), \beta(\equiv \phi_1)$ and $\gamma(\equiv \phi_3)$ can be independently measured [9]. Within the standard model the sum of these three angles is equal to 180°. Thus one tests the standard model by testing whether independent determination of the three angles give consistent results. Over the past decade or so many methods have been proposed for obtaining the three interior angles of the unitarity triangle. In the near future these CP phases will be measured in a variety of experiments at B factories, DESY HERA-B and hadron colliders.

The CP angles are typically extracted from CP violating rate asymmetries in B decays [10]. The phase $\beta \equiv \mathrm{Arg}(-V_{cb}^* V_{cd}/V_{tb}^* V_{td})$ [$=\mathrm{Arg}(V_{td}^*)$, in the standard phase convention] is measured by the time dependent CP asymmetry in $B_d^0(t) \to J/\psi K_S$. Theoretically, this measurement provides a very clean determination of $\sin 2\beta$, since the single phase approximation holds in this case [11,12]. Recently, large CP violation in $B_d^0 \to \psi K_S$ has been observed by the Babar and Belle Collaborations and a clean measurement of β has been made [13,14]. This is the first step towards a serious test of the standard model of CP violation.

The angle $\alpha \equiv \mathrm{Arg}(-V_{tb}^* V_{td}/V_{ub}^* V_{ud})$ can be measured using the CP asymmetries in the decays $B_d^0 \to \pi^+ \pi^-$, however there are theoretical hadronic uncertainties due to the existence of penguin diagrams [11,12]. A theoretical cleaner way of resolving the penguin correction will require the combination of the asymmetry in $B_d^0 \to \pi^+ \pi^-$ with other measurements. A very early suggestion [15] was that one also has to measure the isospin related processes $B^+ \to \pi^+ \pi^0$ and $B^0/\bar{B}^0 \to \pi^0 \pi^0$ and can thereby extract the angle α with reasonable accuracy.

The most difficult to measure is the angle $\gamma \equiv \mathrm{Arg}(-V_{ub}^* V_{ud}/V_{cb}^* V_{cd})$[$=\mathrm{Arg}(V_{ub}^*)$, in the standard convention], the relative weak phase between a Cabibbo-Kobayashi-Maskawa- (CKM-) favored ($b \to c$) and a CKM-suppressed ($b \to u$) decay amplitude. This angle should be measured in a variety of ways so as to check whether one consistently finds the same result. There have been a lot of suggestions and discussions about how to measure this quantity at B factories [16,17]. In Ref. [16] the authors proposed to extract γ using the independent measurements of $B \to D^0 K, B \to \bar{D}^0 K$ and $B \to D_{CP}^0 K$. However, the charged B meson decay mode ($B^- \to \bar{D}^0 K^-$) is difficult to measure experimentally. The reason is that the final $\bar{D}^0$ meson should be identified using $\bar{D}^0 \to K^+ \pi^-$ but it is difficult to distinguish it from the doubly Cabibbo suppressed $D^0 \to K^+ \pi^-$. There are various methods to overcome these difficulties. In Ref. [18] Atwood *et al.* used different final states into which the neutral D meson decays, to extract information on γ. In Ref. [19] Gronau proposed that the angle γ can be determined by using the color allowed decay modes $B^- \to D^0 K^-, B^- \to D_{CP}^0 K^-$ and their charge conjugation modes. In Ref. [20] a new method, using the isospin relations, is suggested to extract γ by exploiting the decay modes $B \to DK^{(*)}$ that are not Cabibbo suppressed. Falk and Petrov [21] recently proposed a new method for measuring γ using the partial rates for CP-tagged B_s decays. It has been discussed in Ref. [22] that it is possible to extract γ cleanly from $B_c \to D^0 D_s$ decays. Sometime ago it was pointed out in [23] that a clean extraction of the angle γ is possible by studying the time dependence of the color allowed decays $B_s^0(\overline{B_s^0}) \to D_s^{\pm} K^{\mp}$.

The angle γ can also be measured using the $SU(3)$ relations between $B \to \pi K, \pi\pi$ decay amplitudes [24,25]. These analyses require additional theoretical input, such as $SU(3)$ flavor symmetry and arguments for the dynamical suppression of rescattering processes. While the validity of some assumptions can be checked in the data itself, these approaches leave theoretical uncertainties, which are hard to quantify reliably. This limits the precision with which they can be used to extract γ by themselves. Nevertheless, they will provide important cross-checks on other techniques as well as help us to address the discrete ambiguities which theoretically cleaner methods leave unresolved.

It has been known for many years now that it is possible to cleanly extract weak phase information using CP violating rate asymmetries in the B system. The earliest studies of such rate asymmetries concentrated on the final states which are CP eigenstates. However, it soon became clear that certain non-CP eigenstates can also be used. It has been shown by Aleksan, Dunietz, Kayser and Le Diberder (ADKL) [26] that the CKM phase can be cleanly determined in the B decays to almost any final state which is accessible to both B_d^0 and $\overline{B_d^0}$. In this paper we will emphasize that the decay modes $B_s^0 \to f$ (where $f = \overline{D^0}\phi$) and $\overline{B_s^0} \to f$ can be used to probe the weak phase γ. In this case sizable CP violating effect can occur because the two interfering amplitudes $B_s^0 \to f$ and $\overline{B_s^0} \to f$ are of comparable size. Another nice feature of the above decay modes is that unique weak phases are involved which arise from tree diagram alone. No contamination from other weak phases are possible. Neither penguin diagrams nor rescattering with different weak phases exist. Hence the extracted weak phase is truely γ. It has been discussed by Gronau and London [27] that it is possible to determine γ considering the time dependent decay rates of the three processes $B_s^0 \to D^0\phi, \overline{D^0}\phi$ and $D_1^0\phi$ (where D_1^0 is a neutral D meson CP eigenstate).

Here, we consider the final state $\overline{D^0}\phi$ and the corresponding vector vector modes $\overline{D}^{*0}\phi$ to which both B_s^0 and $\overline{B_s^0}$ can decay. For the $\overline{D^0}\phi(PV)$ final state we follow the ADKL method [26] and for the vector-vector (VV) final state we use the approach of Ref. [28]. Since currently running e^+e^-B factories operating at the $\Upsilon(4S)$ resonance will not be in a position to explore B_s decays, a strong emphasis has been given to nonstrange B mesons in the recent literature. However, the B_s system also provides interesting strategies to determine γ and can prove to be useful system to understand CP violation. So in order to make use of these methods and explore the CP violation in B_s system, dedicated B physics experiments at hadron machines, such as CERN Large Hadron Collider (LHC), BTeV, etc., are the natural place. Within the standard model, the weak B_s^0-$\overline{B_s^0}$ mixing phase is very small, and studies of B_s decays involve very rapid B_s^0-$\overline{B_s^0}$ oscillations due to large mass difference $\Delta M_s = M_H^s - M_L^s$ between the mass eigenstates. Future B-physics experiments performed at hadron machines should be in a position to resolve the oscillations.

The paper is organized as follows. We present the method for the determination of the angle γ from the decay mode $B_s^0(t)(\overline{B_s^0}(t)) \to \overline{D^0}\phi$ in Sec. II and from $B_s^0(t)(\overline{B_s^0}(t)) \to \overline{D}^{*0}\phi$ in Sec. III. Section IV contains our conclusion.

II. γ FROM $B_S^0(\overline{B_S^0}) \to \overline{D^0}\phi$

Here we consider the final state $\overline{D^0}\phi$ to which both B_s^0 and $\overline{B_s^0}$ can decay. Both the amplitudes proceed via the color suppressed tree diagrams only and there will be no penguin contributions. The amplitude for $B_s^0 \to \overline{D^0}\phi$ arises from the quark transition $\overline{b} \to \overline{c}u\overline{s}$ and has no weak phase in the Wolfenstein parametrization, while the amplitude $\overline{B_s^0} \to \overline{D^0}\phi$ arises from $b \to u\overline{c}s$ and carries the weak phase $e^{-i\gamma}$. The amplitudes also have the strong phases $e^{i\delta_1}$ and $e^{i\delta_2}$. Thus, in general, one can write the decay amplitudes as

$$A(f) = \text{Amp}\ \ (B_s^0 \to \overline{D^0}\phi) = A_1 e^{i\delta_1},$$

$$\overline{A}(f) = \text{Amp}\ \ (\overline{B_s^0} \to \overline{D^0}\phi) = A_2 e^{-i\gamma} e^{i\delta_2}. \qquad (1)$$

The amplitudes for corresponding CP conjugate processes are given as

$$\overline{A}(\overline{f}) = \text{Amp}\ \ (\overline{B_s^0} \to D^0\phi) = A_1 e^{i\delta_1},$$

$$A(\overline{f}) = \text{Amp}\ \ (B_s^0 \to D^0\phi) = A_2 e^{i\gamma} e^{i\delta_2}. \qquad (2)$$

Due to B_s^0-$\overline{B_s^0}$ mixing, a state which is created as a B_s^0 or a $\overline{B_s^0}$ will evolve in time into a mixture of both states [29]. The weak phase of B_s^0-$\overline{B_s^0}$ mixing is decribed by the parameter q/p. Within the standard model, $q/p = (V_{tb}^* V_{ts}/V_{tb} V_{ts}^*)$ is an excellent approximation. In the usual Wolfenstein parametrization it has zero phase and $|q/p| \sim 1$. Since the final state $f = \overline{D^0}\phi$ can be fed from both B_s^0 and $\overline{B_s^0}$, the time dependent rates can be written as [29]

$$\Gamma(B_s^0(t) \to f) = \frac{e^{-\Gamma t}}{2} \Bigg\{ [|A(f)|^2 + |\overline{A}(f)|^2] \cosh\frac{\Delta\Gamma t}{2} + [|A(f)|^2 - |\overline{A}(f)|^2] \cos\Delta m t + 2\,\text{Re}[A(f)^* \overline{A}(f)] \sinh\frac{\Delta\Gamma t}{2} - 2\,\text{Im}[A(f)^* \overline{A}(f)] \sin\Delta m t \Bigg\},$$

$$\Gamma(B_s^0(t)\to\bar{f}) = \frac{e^{-\Gamma t}}{2}\left\{[|\bar{A}(\bar{f})|^2+|A(\bar{f})|^2]\cosh\frac{\Delta\Gamma t}{2} - [|\bar{A}(\bar{f})|^2-|A(\bar{f})|^2]\cos\Delta m t + 2\,\mathrm{Re}[\bar{A}(\bar{f})^* A(\bar{f})]\sinh\frac{\Delta\Gamma t}{2} + 2\,\mathrm{Im}[\bar{A}(\bar{f})^* A(\bar{f})]\sin\Delta m t\right\},$$

$$\Gamma(\bar{B}_s^0(t)\to\bar{f}) = \frac{e^{-\Gamma t}}{2}\left\{[|\bar{A}(\bar{f})|^2+|A(\bar{f})|^2]\cosh\frac{\Delta\Gamma t}{2} + [|\bar{A}(\bar{f})|^2-|A(\bar{f})|^2]\cos\Delta m t + 2\,\mathrm{Re}[\bar{A}(\bar{f})^* A(\bar{f})]\sinh\frac{\Delta\Gamma t}{2} - 2\,\mathrm{Im}[\bar{A}(\bar{f})^* A(\bar{f})]\sin\Delta m t\right\},$$

$$\Gamma(\bar{B}_s^0(t)\to f) = \frac{e^{-\Gamma t}}{2}\left\{[|A(f)|^2+|\bar{A}(f)|^2]\cosh\frac{\Delta\Gamma t}{2} - [|A(f)|^2-|\bar{A}(f)|^2]\cos\Delta m t + 2\,\mathrm{Re}[A(f)^* \bar{A}(f)]\sinh\frac{\Delta\Gamma t}{2} + 2\,\mathrm{Im}[A(f)^* \bar{A}(f)]\sin\Delta m t\right\}, \quad (3)$$

where Γ, Δm and $\Delta\Gamma$ denote the average width, the differences in mass and widths of the heavy and light B_s mesons respectively. If we denote the masses and widths of the two mass eigenstates by $M_{L,H}$ and $\Gamma_{L,H}$ then we have

$$\Gamma = \frac{1}{\tau_{B_s}} = \frac{\Gamma_H+\Gamma_L}{2}, \quad \Delta m = M_H - M_L \quad \text{and} \quad \Delta\Gamma = \Gamma_H - \Gamma_L. \quad (4)$$

Thus the time dependent measurement of $B_s^0(t)\to f$ decay rates allows one to obtain the following observables:

$$|A(f)|^2+|\bar{A}(f)|^2, \quad |A(f)|^2-|\bar{A}(f)|^2, \quad \mathrm{Re}[A(f)^*\bar{A}(f)] \quad \text{and} \quad \mathrm{Im}[A(f)^*\bar{A}(f)]. \quad (5)$$

If $\Delta\Gamma=0$, then only the observables $|A(f)|^2+|\bar{A}(f)|^2$, $|A(f)|^2-|\bar{A}(f)|^2$ and $\mathrm{Im}(A(f)^*\bar{A}(f))$ can be extracted from the time dependent study. However, if $\Delta\Gamma$ is significantly different from zero then all the four observables can be extracted.

Similarly, the time dependent measurements of $B_s^0(t)\to\bar{f}$ decay rates will give another four observables. From these observables, the weak phase γ can be determined, which will be explained below. Now if we substitute the decay amplitudes, defined earlier in Eqs. (1) and (2), in Eq. (3) then we get the decay rates as

$$\Gamma(B_s^0(t)\to f) = \frac{e^{-\Gamma t}}{2}\left\{(|A_1|^2+|A_2|^2)\cosh\frac{\Delta\Gamma t}{2} + (|A_1|^2-|A_2|^2)\cos\Delta m t + 2A_1A_2\cos(\delta-\gamma)\times\sinh\frac{\Delta\Gamma t}{2} - 2A_1A_2\sin(\delta-\gamma)\sin\Delta m t\right\},$$

$$\Gamma(B_s^0(t)\to\bar{f}) = \frac{e^{-\Gamma t}}{2}\left\{(|A_1|^2+|A_2|^2)\cosh\frac{\Delta\Gamma t}{2} - (|A_1|^2 - |A_2|^2)\cos\Delta m t + 2A_1A_2\cos(\delta+\gamma)\times\sinh\frac{\Delta\Gamma t}{2} + 2A_1A_2\sin(\delta+\gamma)\times\sin\Delta m t\right\},$$

$$\Gamma(\bar{B}_s^0(t)\to\bar{f}) = \frac{e^{-\Gamma t}}{2}\left\{(|A_1|^2+|A_2|^2)\cosh\frac{\Delta\Gamma t}{2} + (|A_1|^2-|A_2|^2)\cos\Delta m t + 2A_1A_2\cos(\delta+\gamma)\times\sinh\frac{\Delta\Gamma t}{2} - 2A_1A_2\sin(\delta+\gamma)\times\sin\Delta m t\right\},$$

$$\Gamma(\bar{B}_s^0(t)\to f) = \frac{e^{-\Gamma t}}{2}\left\{(|A_1|^2+|A_2|^2)\cosh\frac{\Delta\Gamma t}{2} - (|A_1|^2-|A_2|^2)\cos\Delta m t + 2A_1A_2\cos(\delta-\gamma)\times\sinh\frac{\Delta\Gamma t}{2} + 2A_1A_2\sin(\delta-\gamma)\times\sin\Delta m t\right\}, \quad (6)$$

where $\delta=\delta_2-\delta_1$ is the strong phase difference between the two amplitudes $\bar{B}_s^0\to f$ and $B_s^0\to f$. Thus through the measurements of the time dependent rates, it is possible to measure the amplitudes A_1 and A_2 and the CP violating quantities $S\equiv\sin(\delta+\gamma)$ and $\bar{S}\equiv\sin(\delta-\gamma)$. In turn these quantities will determine $\sin^2\gamma$ up to a fourfold ambiguity via the expression

$$\sin^2\gamma = \frac{1}{2}[1-S\bar{S}\pm\sqrt{(1-S^2)(1-\bar{S}^2)}). \quad (7)$$

Here one of the signs on the right-hand side gives the true $\sin^2\gamma$, while the other gives $\cos^2\delta$. Thus $\sin^2\gamma$ can be extracted cleanly with no hadronic uncertainties but with four fold discrete ambiguity. If the two mass eigenstates have widths which differ enough to result in measurable effect, it becomes possible to experimentally resolve some of the ambiguities in the determination of $\sin^2\gamma$.

Alternatively, one can also determine $\sin 2\gamma$ up to fourfold ambiguity using the relation

$$\sin 2\gamma = (S\sqrt{(1-\bar{S}^2)} - \bar{S}\sqrt{(1-S^2)}). \tag{8}$$

Thus the measurements allow one to determine both $\sin^2\gamma$ and $\sin 2\gamma$ and hence to extract γ unambiguously.

Now let us compute the magnitudes of the amplitudes $A(f)$ and $\bar{A}(f)$ and the corresponding branching ratios for such modes. Using the factorization assumption the amplitudes for these modes are given as

$$\begin{aligned} A(f) &= \mathrm{Amp}(B_s^0(P) \to \bar{D}^0(q)\phi(\epsilon,p)) \\ &= \frac{G_F}{\sqrt{2}} V_{cb}^* V_{us}\ a_2\ f_D\ 2m_\phi A_0(m_D^2)(\epsilon^*\cdot q). \end{aligned} \tag{9}$$

It should be noted here that the nonfactorizable contributions play a significant role in color suppressed B decays. However, since we are interested in finding out the order of magnitudes of the decay amplitudes, here we consider only the factorizable contributions. So in general, one can expect that the estimated branching ratios may be either enhanced or reduced due to the nonfactorizable effects.

The decay amplitude for the process $\bar{B}_s^0 \to \bar{D}^0\phi$ can be found by substituting the appropriate CKM matrix elements in Eq. (9). Hence, one can write

$$\begin{aligned} |\bar{A}(f)/A(f)| &= |\mathrm{Amp}(\bar{B}_s^0 \to \bar{D}^0\phi)/\mathrm{Amp}(B_s^0 \to \bar{D}^0\phi)| \\ &= |V_{ub}V_{cs}^*/V_{cb}^*V_{us}| \sim 0.4. \end{aligned} \tag{10}$$

Thus the advantage of using this final state is that since the two interfering amplitudes are of comparable size the CP violating asymmetry will be large. The decay rates are given as

$$\Gamma(B_s^0 \to \bar{D}^0\phi) = \frac{G_F^2}{4\pi}|V_{cb}^*V_{us}|^2 a_2^2 f_D^2 |A_0(m_D^2)|^2 |p|^3. \tag{11}$$

The value of the form factor at zero momentum transfer, i.e., $A_0(0)$ can be found from Bauer-Stech-Wirbel (BSW) model [30] with value $A_0(0)=0.272$ [31]. The momentum dependence of A_0 can be found out assuming nearest pole dominance given as

$$A_0(q^2) = \frac{A_0(0)}{1-q^2/m_p^2}, \tag{12}$$

where the value of the pole mass $m_p = 5.38$ GeV. Using $f_D = 300$ MeV and $a_2 = 0.3$, the branching ratio is found to be

$$Br(B_s^0 \to \bar{D}^0\phi) = 1.65\times 10^{-5}. \tag{13}$$

Similarly using Eqs. (10) and (13), the branching ratio for $\bar{B}_s^0 \to \bar{D}^0\phi$ is found to be

$$Br(\bar{B}_s^0 \to \bar{D}^0\phi) = 2.64\times 10^{-6}. \tag{14}$$

The predicted branching ratios for these decay modes, which are of the order of $\sim(10^{-5}-10^{-6})$ imply that they can easily be accessible in the second generation hadronic B factories. In hadron machines such as run II of Tevatron, LHC, etc., one anticipates a very large data sample of B_s mesons.

We now make a crude estimate of the number of events required to measure γ by this method. An estimate of the sensitivity of the method can be obtained by comparing it for example to $B_s^0 \to D_s^- K^+$ decay. The BTeV experiment with luminosity $2\times 10^{32}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$ will produce 5×10^{10} number of $B_s^0\bar{B}_s^0$ pairs per 10^7 sec of running time. The expected number of $B_s^0 \to D_s K$ events produced per year is 13100 [32]. The branching ratio for $B_s^0 \to D_s^- K^+$ is 2×10^{-4}, while the branching ratio for $B_s^0 \to \bar{D}^0\phi$ is one order magnitude smaller. If we take the branching ratios as $BR(B_s^0 \to \bar{D}^0\phi) \sim 10^{-5}$, $BR(\bar{D}^0 \to K^+\pi^-$ and $K^+\pi^-\pi^+\pi^-)$ to be 0.12 and $BR(\phi \to K^+K^-)=0.5$, the reconstruction efficiency to be 0.05, and the trigger effeciency level as 0.9, we expect around 1350 number of reconstructed $B_s^0 \to \bar{D}^0\phi$ events per year. An important issue of using this method is that the tagging of the initial B_s meson is required. If we assume the tagging efficiency to be 0.70 [32] we expect approximately 945 tagged events per year. For the LHC experiments also several tagging strategies have already been studied successfully [33]. It is expected that a complete description of the tagging studies will be available by the time the second generation hadron machines are in operation. Thus the question of whether this analysis is possible rests on whether the time resolution is sufficient to separate the three different time-dependent terms.

III. γ FROM $B_s^0(\bar{B}_s^0) \to \bar{D}^{*0}\phi$

Now we consider the final state $f = \bar{D}^{0*}\phi$, consisting of two vector mesons to which both B_s^0 and $\bar{B}_s^0$ can decay. Because the final state does not have a well defined angular momentum, the final state $\bar{D}^{*0}\phi$ cannot be a CP eigenstate. By examining the decay products of $\bar{D}^{*0}\phi$, one can measure the various helicity components of the final state. Since each helicity state corresponds to a well defined CP, an angular analysis of $B_s^0 \to \bar{D}^{*0}\phi$, allows one to extract the CKM phase cleanly. Here we will follow similar approach to that of London *et al.* [28] for the extraction of γ. Due to the interference between different helicity states, there are enough independent observables for the decays of B_s^0 and $\bar{B}_s^0$ to the common final state $\bar{D}^{*0}\phi$, from which the angle γ can be extracted cleanly. If both the final state mesons subsequently decay into two $J^P=0^-$ mesons, i.e., $\bar{D}^{*0} \to K^+\pi^-$ and $\phi \to K^+K^-$, the amplitude can be expressed in the linear polarization basis ($A_\parallel, A_\perp$ and A_0) [34] as

$$\mathcal{A}=\text{Amp}(B_s^0\to f)=A_0x_0+A_{\|}x_{\|}+iA_{\perp}x_{\perp}\,,$$
$$\mathcal{A}'=\text{Amp}(\overline{B}_s^0\to f)=A_0'x_0+A_{\|}'x_{\|}-iA_{\perp}'x_{\perp}\,,\tag{15}$$

where $x_\lambda(\lambda=0,\|,\perp)$ are the coefficients of the helicity amplitudes in the linear polarization basis, depend only on the angles describing the kinematics [34].

Using CPT invariance one can also write the amplitudes for the corresponding CP conjugate processes as

$$\overline{\mathcal{A}}=\text{Amp}(\overline{B}_s^0\to\overline{f})=\overline{A}_0x_0+\overline{A}_{\|}x_{\|}-i\overline{A}_{\perp}x_{\perp}\,,$$
$$\overline{\mathcal{A}}'=\text{Amp}(B_s^0\to\overline{f})=\overline{A}_0'x_0+\overline{A}_{\|}'x_{\|}+i\overline{A}_{\perp}'x_{\perp}\,.\tag{16}$$

With the above Eqs. (15) and (16) the time dependent decay rate $B_s^0(t)\to f$ can be written as

$$\Gamma(B_s^0(t)\to f)=e^{-\Gamma t}\sum_{\lambda\leqslant\sigma}(X_{\lambda\sigma}+Y_{\lambda\sigma}\cos\Delta mt-Z_{\lambda\sigma}\sin\Delta mt)x_\lambda x_\sigma\,.\tag{17}$$

Thus, by performing a time dependent study and angular analysis of the decay $B_s^0(t)\to f$, one can measure the observables $X_{\lambda\sigma}$, $Y_{\lambda\sigma}$ and $Z_{\lambda\sigma}$. In terms of the helicity amplitudes these observables can be expressed as follows:

$$X_{\lambda\lambda}=\frac{|A_\lambda|^2+|A_\lambda'|^2}{2},\quad Y_{\lambda\lambda}=\frac{|A_\lambda|^2-|A_\lambda'|^2}{2},$$
$$X_{\perp i}=-\text{Im}(A_\perp A_i^*-A_\perp' A_i'^*),$$
$$X_{\|0}=\text{Re}(A_{\|}A_0^*+A_{\|}'A_0'^*),$$
$$Y_{\perp i}=-\text{Im}(A_\perp A_i^*+A_\perp' A_i'^*),$$
$$Y_{\|0}=\text{Re}(A_{\|}A_0^*-A_{\|}'A_0'^*)$$
$$Z_{\perp i}=-\text{Re}(A_\perp^*A_i'+A_i^*A_\perp'),\quad Z_{\perp\perp}=-\text{Im}(A_\perp^*A_\perp'),$$
$$Z_{\|0}=\text{Im}(A_{\|}^*A_0'+A_0^*A_{\|}'^*),\quad Z_{ii}=\text{Im}(A_i^*A_i'),\tag{18}$$

where $i=\{0,\|\}$. Similarly, the decay rate for $B_s^0\to\overline{f}$ can be given as

$$\Gamma(B_s^0(t)\to f)=e^{-\Gamma t}\sum_{\lambda\leqslant\sigma}(\overline{X}_{\lambda\sigma}+\overline{Y}_{\lambda\sigma}\cos\Delta mt-\overline{Z}_{\lambda\sigma}\sin\Delta mt)x_\lambda x_\sigma\,.\tag{19}$$

The expressions for the observables $\overline{X}_{\lambda,\sigma},\overline{Y}_{\lambda,\sigma}$ and $\overline{Z}_{\lambda,\sigma}$ are similar to those given in Eq. (18) with the replacements $A_\lambda\to\overline{A}_\lambda'$ and $A_\lambda'\to\overline{A}_\lambda$.

For the purpose of the exctraction of γ let us first write the helicity amplitudes for each processes. Since they are described by the color suppressed tree diagrams, only a single CKM weak phase will be involved in these amplitudes and are given as

$$A_\lambda=\text{Amp}(B_s^0\to f)_\lambda=a_\lambda e^{i\delta_\lambda^1},$$
$$A_\lambda'=\text{Amp}(\overline{B}_s^0\to f)_\lambda=b_\lambda e^{-i\gamma}e^{i\delta_\lambda^2},$$
$$\overline{A}_\lambda'=\text{Amp}(B_s^0\to\overline{f})_\lambda=b_\lambda e^{i\gamma}e^{i\delta_\lambda^2},$$
$$\overline{A}_\lambda=\text{Amp}(\overline{B}_s^0\to\overline{f})_\lambda=a_\lambda e^{i\delta_\lambda^1},\tag{20}$$

where $\gamma=\text{Arg}(V_{ub}^*)$ represents the weak phase and $\delta_\lambda^1,\delta_\lambda^2$ are the strong phases. With these above expressions for the various amplitudes, we now show how to extract the weak phase γ using the decay rate measurements. It is now very easy to see that the observables can be written in terms of the helicity amplitudes as

$$X_{\lambda\lambda}=\overline{X}_{\lambda\lambda}=\frac{|a_\lambda|^2+|b_\lambda|^2}{2},\quad Y_{\lambda\lambda}=-\overline{Y}_{\lambda\lambda}=\frac{|a_\lambda|^2-|b_\lambda|^2}{2}.\tag{21}$$

Thus, one can determine the magnitudes of various helicity amplitudes $|a_\lambda|^2$ and $|b_\lambda|^2$ from Eq. (21). Next we obtain the expressions for the observables : $X_{\perp i}$, $Y_{\perp i}$, $X_{\|0}$ and $Y_{\|0}$,

$$X_{\perp i}=-\overline{X}_{\perp i}=b_\perp b_i\sin(\delta_\perp+\Delta_i-\delta_i)-a_\perp a_i\sin\Delta_i\,,$$
$$Y_{\perp i}=\overline{Y}_{\perp i}=-b_\perp b_i\sin(\delta_\perp+\Delta_i-\delta_i)-a_\perp a_i\sin\Delta_i\,,\tag{22}$$

where $\Delta_i=\delta_\perp^1-\delta_i^1$ and $\delta_\lambda=\delta_\lambda^2-\delta_\lambda^1$. Using Eq. (22) one can solve for $a_\perp a_i\sin\Delta_i$. Similarly, one can write

$$X_{\|0}=\overline{X}_{\|0}=b_{\|}b_0\cos(\delta_{\|}+\Delta-\delta_0)+a_{\|}a_0\cos\Delta,$$
$$Y_{\|0}=-\overline{Y}_{\|0}=-b_{\|}b_0\cos(\delta_{\|}+\Delta-\delta_0)+a_{\|}a_0\cos\Delta,\tag{23}$$

where $\Delta=\delta_{\|}^1-\delta_0^1$. Thus one can solve for $a_{\|}a_0\cos\Delta$ using Eq. (23).

The coefficients of sin(Δmt) term, which can be obtained in a time dependent study, can be written as

$$Z_{ii}=a_ib_i\sin(\delta_i-\gamma),\quad \overline{Z}_{ii}=-a_ib_i\sin(\delta_i+\gamma),$$
$$Z_{\perp\perp}=-a_\perp b_\perp\sin(\delta_\perp-\gamma),\quad \overline{Z}_{\perp\perp}=a_\perp b_\perp\sin(\delta_\perp+\gamma).\tag{24}$$

Thus we can find

$$2b_i\cos\delta_i=-\frac{Z_{ii}+\overline{Z}_{ii}}{a_i\sin\gamma},\quad 2b_i\sin\delta_i=\frac{Z_{ii}-\overline{Z}_{ii}}{a_i\cos\gamma},$$
$$2b_\perp\cos\delta_\perp=\frac{Z_{\perp\perp}+\overline{Z}_{\perp\perp}}{a_\perp\sin\gamma},$$

$$2b_\perp \sin\delta_\perp = -\frac{Z_{\perp\perp}-\overline{Z}_{\perp\perp}}{a_\perp \cos\gamma}. \tag{25}$$

Similarly, the terms involving interference of different helicities are given as

$$Z_{\perp i} = -[a_\perp b_i \cos(\delta_i - \Delta_i - \gamma) + a_i b_\perp \cos(\delta_\perp + \Delta_i - \gamma)],$$

$$\overline{Z}_{\perp i} = -[a_\perp b_i \cos(\delta_i - \Delta_i + \gamma) + a_i b_\perp \cos(\delta_\perp + \Delta_i + \gamma)] \tag{26}$$

and

$$Z_{\|0} = [a_\| b_0 \sin(\delta_0 - \Delta - \gamma) + a_0 b_\| \sin(\delta_\| + \Delta - \gamma)],$$

$$\overline{Z}_{\|0} = -[a_\| b_0 \sin(\delta_0 - \Delta + \gamma) + a_0 b_\| \sin(\delta_\| + \Delta + \gamma)]. \tag{27}$$

Considering all the above information together, we are now in a position to extract the weak phase γ. Making use of Eq. (25) we can rewrite Eq. (26) in the following two useful forms:

$$Z_{\perp i} + \overline{Z}_{\perp i} = a_i a_\perp \cos\Delta_i \cot\gamma \left[\frac{Z_{ii}+\overline{Z}_{ii}}{a_i^2} - \frac{Z_{\perp\perp}+\overline{Z}_{\perp\perp}}{a_\perp^2}\right] - a_i a_\perp \sin\Delta_i \left[\frac{Z_{ii}-\overline{Z}_{ii}}{a_i^2} + \frac{Z_{\perp\perp}-\overline{Z}_{\perp\perp}}{a_\perp^2}\right], \tag{28}$$

$$Z_{\perp i} - \overline{Z}_{\perp i} = -a_i a_\perp \cos\Delta_i \tan\gamma \left[\frac{Z_{ii}-\overline{Z}_{ii}}{a_i^2} - \frac{Z_{\perp\perp}-\overline{Z}_{\perp\perp}}{a_\perp^2}\right] - a_i a_\perp \sin\Delta_i \left[\frac{Z_{ii}+\overline{Z}_{ii}}{a_i^2} + \frac{Z_{\perp\perp}+\overline{Z}_{\perp\perp}}{a_\perp^2}\right]. \tag{29}$$

Now let us closely look into the terms involved in the above two Eqs. (28) and (29). We already know most of the terms: (i) $Z_{\lambda\lambda}$, $\overline{Z}_{\lambda\lambda}$ are measurable quantities. (ii) a_λ^2 are known quantities and can be determined from Eq. (21). (iii) $a_i a_\perp \sin\Delta_i$ is obtained from Eq. (22). Thus, these two equations involve only two unknown quantities $\tan\gamma$ and $a_i a_\perp \cos\Delta_i$ and hence can be easily solved upto a sign ambiguity in each of these quantities. Thus $\tan^2\gamma$ or equivalently $\sin^2\gamma$ can be obtained from the angular analysis.

Similarly, if we put all the informations in Eq. (27) we obtain the following relations:

$$Z_{\|0} + \overline{Z}_{\|0} = a_0 a_\| \cos\Delta \cot\gamma \left[\frac{Z_{00}+\overline{Z}_{00}}{a_0^2} + \frac{Z_{\|\|}+\overline{Z}_{\|\|}}{a_\|^2}\right] - a_\| a_0 \sin\Delta \left[\frac{Z_{00}-\overline{Z}_{00}}{a_0^2} - \frac{Z_{\|\|}-\overline{Z}_{\|\|}}{a_\|^2}\right], \tag{30}$$

$$Z_{\|0} - \overline{Z}_{\|0} = a_0 a_\| \cos\Delta \left[\frac{Z_{00}-\overline{Z}_{00}}{a_0^2} + \frac{Z_{\|\|}-\overline{Z}_{\|\|}}{a_\|^2}\right] + a_\| a_0 \sin\Delta \cot\gamma \left[\frac{Z_{00}+\overline{Z}_{00}}{a_0^2} - \frac{Z_{\|\|}+\overline{Z}_{\|\|}}{a_\|^2}\right]. \tag{31}$$

In the above two Eqs. (30) and (31) we know all the quantities except two, i.e., $a_\| a_0 \cos\Delta$ and $\cot\gamma$. Thus these two unknowns can easily be determined from these two equations. Thus one can solve for $\tan^2\gamma$ or equivalently $\sin^2\gamma$ without any hadronic uncertainties.

To calculate the number of events necessary to determine γ using this method we have to know the branching ratios for the decay modes $B_s^0(\overline{B}_s^0) \to \overline{D^{*0}}\phi$. The decay widths (in units of 10^{12} s^{-1}) for these decay modes are calculated in Ref. [35] as

$$\Gamma(B_s^0 \to \overline{D}^{*0}\phi) = 2\times a_2^2 |V_{cb}^* V_{us}|^2,$$

$$\Gamma(\overline{B}_s^0 \to \overline{D}^{*0}\phi) = 1.9 a_2^2 |V_{ub} V_{cs}^*|^2. \tag{32}$$

Thus the branching ratios are given as

$$BR(B_s^0 \to \overline{D}^{*0}\phi) = 2.09\times 10^{-5},$$

$$BR(\overline{B}_s^0 \to \overline{D}^{*0}\phi) = 3.61\times 10^{-6}. \tag{33}$$

Thus if we assume the same selection method as we have done for $B_s^0 \to \overline{D}^0\phi$ case, here we get approximately 2700 reconstructed $B_s^0 \to \overline{D}^{*0}\phi$ events per year of running at BTeV. Assuming the tagging efficiency to be 0.70, one expects approximately 1890 tagged events per year.

IV. CONCLUSION

In this paper we have discussed the determination of the CKM phase γ from time dependent measurement of the pure tree nonleptonic B_s^0 decay modes $B_s^0(t)(\overline{B}_s^0(t)) \to \overline{D}^0\phi$ and $B_s^0(t)(\overline{B}_s^0(t))) \to \overline{D}^{*0}\phi$. For the former case we have used the formalism similar to that of ADKL method [26] and for the latter case we have followed the approach of London *et al.* [28]. The advantage of these decay modes is that they are described by pure tree diagrams and hence free from theoretical hadronic uncertainties. Within the standard model these modes are expected to exhibit branching ratios at the $10^{-5}-10^{-6}$ level. So they are expected to be easily accessible in the second generation hadronic B factories.

The accurate determination of phase γ of the unitarity triangle is really challenging. Therefore, one should study as many decay modes as possible to cross check other findings and also explore various strategies for the clean determination of it.

For the case of (PV) final states i.e, $B_s^0(\overline{B}_s^0) \to \overline{D}^0\phi$ we have obtained four observables (A_1, A_2, S and $\overline{S}$), from the corresponding time dependent decay rates. From these observables γ can be extracted using Eq. (7) or (8). Furthermore, one can also find the information about strong phase [Eq. (7)] from this analysis. Next, we have considered the (VV) final states $B_s^0(\overline{B}_s^0) \to \overline{D}^{*0}\phi$. We have used linear polarization basis to write the decay rates in terms of the observables (X,Y,Z). From these observables, one can exctract γ solving Eqs. (28), (29) or Eqs. (30), (31). It should be emphasized here that using our analysis, the extraction of γ can be done cleanly without any hadronic uncertainties in both the cases of PV and VV final states of B_s meson, but with some amout of discrete ambiguities. Furthermore, these modes may possibly guide us to know physics beyond standard model and/or valuable informations regarding the nature of CP violation.

To summarize, we point out here that it is indeed possible to determine the weak phase γ cleanly from the time dependent measurement of the nonleptonic decay modes $B_s^0(\overline{B}_s^0) \to \overline{D}^0\phi$ and the corresponding vector vector modes $B_s^0(\overline{B}_s^0) \to \overline{D}^{*0}\phi$. The strategies presented in this paper appear to be particularly interesting for second generation experiments at hadron machines such as LHC and BTeV, where also the very powerful physics potential of the B_s meson can be exploited.

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